THE ISING MODEL AND BUBBLES IN THE QUARK–GLUON PLASMA[a]

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I review evidence for the stability of bubbles in the quark–gluon plasma near the confinement phase transition. In analogy with the much-studied oil–water emulsions, this raises the possibility that there are many phases between the pure plasma and the pure hadron gas, characterized by spontaneous inhomogeneity and modulation. In studying emulsions, statistical physicists have reproduced many of their phases with microscopic models based on Ising-like theories with competing interactions. Hence we seek an effective Ising Hamiltonian for the SU(3) gauge theory near its transition.

1 Background

I report here on efforts by Nathan Weiss and myself to construct an Ising effective Hamiltonian for the SU(3) gauge theory near its confinement phase transition.[1] I will devote most of the talk to presenting the motivation for our work. Its origin is in hints of strange doings in the physics of bubbles in the quark–gluon plasma, hints that raise the possibility of a wealth of phases in the neighborhood of the confinement transition.

1.1 Bubbles in the plasma

The bag model provided the first hint of unconventional physics associated with bubbles in the quark–gluon plasma. Mardor and Svetitsky[2] calculated the free energy F of a bubble of radius R containing a pion gas, surrounded by plasma, at temperatures near the transition. The result, shown in Fig. 1, is that $F(R)$ just above the transition has a minimum at non-zero R. This means that a bubble, instead of entirely shrinking away above the transition, will shrink to a fixed radius and stay there; moreover, each bubble lowers the total free energy of the system, so that many bubbles will form spontaneously, limited only by the (unknown) bubble–bubble interaction energy. The pure plasma is thus unstable against the formation of a *Swiss cheese*.[3]

Looking a bit more deeply, Mardor and Svetitsky considered an expansion

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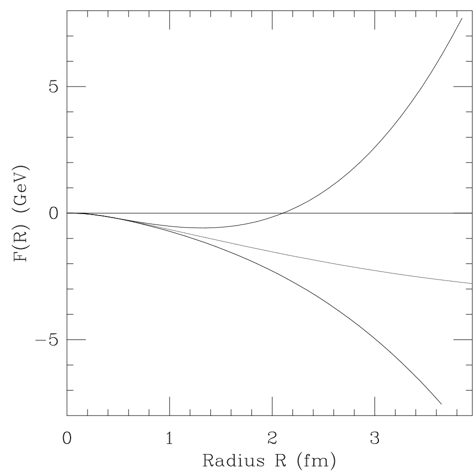

Figure 1: Free energy of a bubble in the plasma, calculated in the bag model. The three curves are, top to bottom, for temperatures just above, at, and just below the transition.

in inverse powers of R, beginning with volume, surface, and "curvature" terms,

$$F = \Delta P \cdot \frac{4}{3}\pi R^3 + \sigma \cdot 4\pi R^2 + \alpha \cdot 8\pi R + \cdots \ . \tag{1}$$

A local version of Eq. 1 applies to an interface of arbitrary shape:

$$F = \Delta P\, V + \sigma \int dS + \alpha \int dS \left(\frac{1}{R_1} + \frac{1}{R_2} \right) + \cdots \ , \tag{2}$$

where the last term shown is an integral of the extrinsic curvature, specified by the local principal radii. The bag result is that (in either equation) σ is small (compared to the natural scale $B^{3/4}$) while α is not, and furthermore that $\alpha < 0$ for hadron bubbles in the plasma.[b]

All this could be dismissed as an oddity of the bag model until lattice calculations were done. So far only the pure gauge theory has been studied, but indeed the surface tension[4] σ turns out to be perhaps two orders of magnitude smaller than expected ($\sigma \simeq 0.02\, T_c^3$) and a study of actual spherical bubbles[4] showed a negative curvature coefficient α and perhaps even a non-trivial minimum in $F(R)$. It is worth noting that the bubble interface on the lattice is quite thick, so these lattice calculations go beyond the handwaving associated with the thin wall of the bag model.

[b] α flips sign and becomes positive for plasma droplets in the hadron gas.

1.2 Oil–water emulsions

Instead of a mixture of the low- and high-temperature phases of QCD, consider a more down-to-earth mixture of water and oil.[5] The relative amounts can be controlled via the two fluids' chemical potentials; the interface area between the two can be controlled by adding soap, with its own chemical potential. The interface acquires a spontaneous curvature, which can be adjusted by adding salt to the water.

One approach to the study of such mixtures is to write the free energy of a single interface as an expansion of the form of Eq. 2 (with additional terms, quadratic in the curvature, to ensure stability). A large variety of phases has been demonstrated in this way, including global separation of the oil and water; spherical bubbles; cylindrical bubbles; planar lamellae; and interpenetrating percolation networks. One may also write down *local* models for these systems,[6] typically by defining a spin variable σ which is ±1 for oil and water. An effective Hamiltonian might be

$$H_{\mathrm{eff}} = J \sum_{\langle ij \rangle} \sigma_i \sigma_j + h \sum \sigma_i + \gamma \sum_{\mathrm{nnn}} \sigma_i \sigma_k + \delta \sum \sigma_i \sigma_j \sigma_k \ , \tag{3}$$

with a negative (*i.e.,* ferromagnetic) nearest-neighbor coupling J and a *competing,* positive (antiferromagnetic) next-nearest-neighbor coupling γ. The competition between couplings leads to long-range domain structure, and the odd terms contribute spontaneous curvature to the interfaces. If we can derive an effective Hamiltonian of this form for the SU(3) gauge theory, we can see whether it possesses the competing interactions needed to establish domain structure in equilibrium.

2 Effective Hamiltonian for SU(3) gauge theory

We map configurations of the $d = 4$ SU(3) gauge theory to Ising spins in 3 dimensions $\sigma = \pm1$ as follows. We identify confining and non-confining domains according to the local value of the Wilson line,

$$L(\mathbf{x}) = \mathrm{Tr}\, \mathrm{P}\, \exp i \int_0^\beta dt\, A_0(\mathbf{x}, t) \ , \tag{4}$$

with $\sigma(\mathbf{x}) = +1$ if $|L(\mathbf{x})| > r_\sigma$ (with r_σ suitably chosen) and $\sigma = -1$ otherwise. In order to obtain clear separation between confining and non-confining domains, we smear L over a $2 \times 2 \times 2$ block before mapping it to σ, as shown in Fig. 2. Configurations of L, generated by Monte Carlo simulation of the

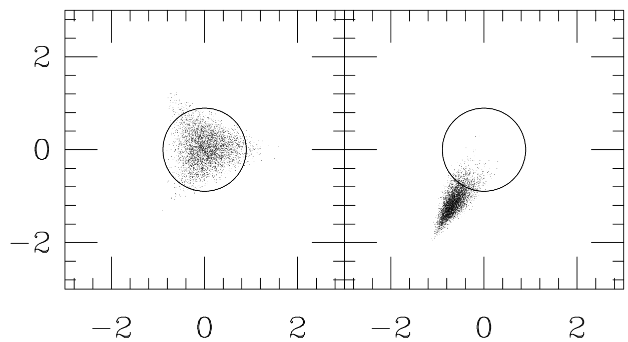

Figure 2: Distribution of $L(\mathbf{x})$ in the complex plane, after averaging over blocks of 8 sites, in the confining phase (left) and in the non-confining phase (right). Circles have radius $r_\sigma^2 = 0.8$.

SU(3) gauge theory, thus yield configurations of σ; these in turn yield an effective Hamiltonian $H_{\rm eff}[\sigma]$ along the lines of Eq. 3 via solution of the lattice Schwinger-Dyson equations.[7]

Coefficients in $H_{\rm eff}[\sigma]$ are shown in the table. The first two lines are the couplings for the theory at the phase transition, $\beta = 5.091$ on our lattice, when it is approached from the ordered and disordered sides, respectively. L has been smeared, as noted, and a $2 \times 16 \times 16 \times 16$ lattice gauge theory has been mapped to a $16 \times 16 \times 16$ Ising theory. We note competition between the ferromagnetic nn coupling and the antiferromagnetic $4^{\rm th}$-neighbor (two-link) coupling, but the latter is uncomfortably strong, indicating that perhaps more couplings should be retained.[c] The next two lines in the table result upon decimating the spins, giving an $8 \times 8 \times 8$ Ising theory. The couplings are short-ranged, but competition has disappeared.

In both cases the Ising couplings are discontinuous as we cross the phase transition. This is inconsistent with the expectation that we should obtain a single Ising theory whose first-order phase boundary produces the confinement phase transition. It is also inconsistent with a theorem proved by van Enter, Fernández, and Sokal,[8] which states (in brief) that effective Hamiltonians are either continuous or nonsense. It is possible that $H_{\rm eff}[\sigma]$ may be made continuous by adding (many) more couplings to it; it is also possible that there exists *no* effective Ising Hamiltonian with sensible interactions. If the latter

[c]In fact the Ising model with these couplings gives a fairly poor match to the expectation values in the gauge theory.

Table 1: Couplings for the effective Ising Hamiltonian. The first two lines reflect smearing of L; the last two lines reflect smearing and decimation.

	h	nn	nnn	$3^{\rm rd}$	$4^{\rm th}$	3-spin$_1$	3-spin$_2$
ordered:	-0.05	-0.46	-0.05	0.04	0.15	-0.006	0.006
disordered:	0.14	-0.39	-0.03	0.05	0.13	0.02	0.02
ordered:	0.03	-0.13	-0.02	-0.006	0.002	0.004	-0.003
disordered:	0	-0.24	-0.05	0	0	-0.02	0

should prove true, a possible solution lies in defining a more complex effective spin, perhaps incorporating a $Z(3)$ degree of freedom in order to preserve the symmetry of the original gauge theory.[9]

Acknowledgments

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